LITERATURE CITED

1. V. P. Stulov, "Equations of a laminar boundary layer in a two-phase medium," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 1 (1979).
2. A. N. Osiptsov, "Structure of the laminar boundary layer of a disperse mixture on a flat plate," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 4 (1980).
3. Yu. M. Tsirkunov, "Effect of a viscous boundary layer on particle deposition in the flow of a gas suspension about a sphere," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 1 (1982).
4. M. Van Dyke, Perturbation Methods in Fluid Mechanics, Parabolic Press (1975).
5. L. A. Klebanov, L. E. Kroshilin, et al., "Hyperbolicity, stability, and correction of a Cauchy problem for the system of equations of two-velocity motion of two-phase media," Prikl. Mat. Mekh., 46, No. 1 (1982).
6. A. N. Kraiko, "Two-fluid model of the flow of a gas and particles dispersed in the gas," Prikl. Mat. Mekh., 46, No. 1 (1982).
7. I. V. Petukhov, "Numerical calculation of two-dimensional flows in a boundary layer," in: Numerical Methods of Solving Differential and Integral Equations and Quadrature Formulas [in Russian], Nauka, Moscow (1964).
8. V. M. Voloshchuk, Introduction to the Hydrodynamics of Coarsely-Dispersed Aerosols [in Russian], Gidrometeoizdat, Leningrad (1971).
9. H. Schlichting, Boundary Layer Theory, 7th ed., McGraw-Hill (1979).
10. A. N. Kraiko, "Surfaces of discontinuity in a medium devoid of internal pressure," Prikl. Mat. Mekh., 43, No. 3 (1979).
11. G. A. Tirskii, "Conditions for surfaces of nonremovable discontinuity in a multicomponent mixture," Prikl. Mat. Mekh., 25, No. 2 (1961).

INFLUENCE OF AN ABRUPT CHANGE IN THE THERMAL BOUNDARY CONDITIONS ON THE TURBULENT BOUNDARY LAYER ON A PLATE

L. N. Drozdova and A. L. Sorokin

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The development of the thermal boundary layer within a dynamic layer that has already been formed is a situation that is often encountered in the practice of analyzing heat exchangers. The formulation of this problem is represented schematically in Fig. 1. A homogeneous thermal flux q_W acts on a plate with section $x = x_0$ (x_0 is the length of the unheated section), or the surface temperature changes to T_W by a "jump." Here δ_0 is the thickness of the dynamic boundary layer in the section of the "jump," L is the length of the heat transfer section, and u_e and T_e are the rate and temperature of the main flow. The flow is quasiisothermal. This problem is solved by integral methods in [1, 2]. But this approach is inadequately general since it requires additional empirical information. Moreover, it is difficult to obtain a detailed flow pattern by the integral method.

The Patankar–Spalding finite-difference method of solving the system of boundary layer differential equations is used in this paper to solve the formulated problem. The method underestimates the value of the Stanton number St, especially near the section $x = x_0$, where the discrepancy between the experimental results and a computation is about 40% for the data from [2] and about 15% for data from [3].

The method mentioned was also applied by other authors [4] to solve an analogous problem. They visibly experienced similar difficulties since they selected the turbulent Prandtl number Pr_T over the boundary layer section to obtain agreement between experiment and theory in the "jump" zone. The computation was performed with the variable

$$\mathrm{Pr}_\mathrm{T}(y) = \frac{\varkappa\,[1 - \exp(-y/A)]}{\varkappa_k\,[1 - \exp(-y/B)]}, \tag{1}$$

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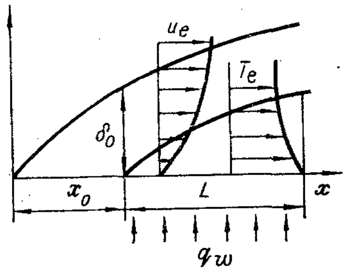

Fig. 1

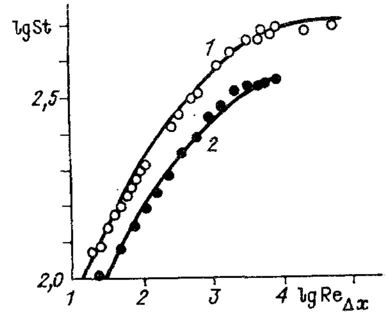

Fig. 2

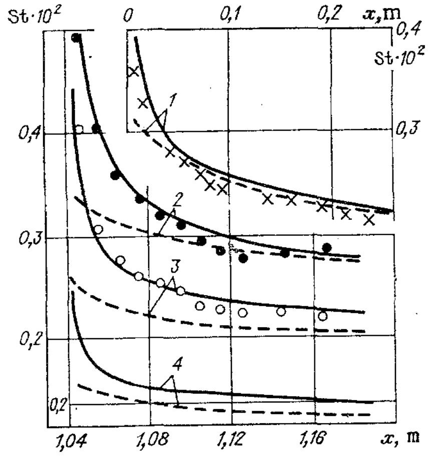

Fig. 3

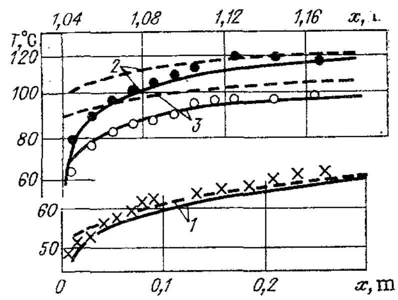

Fig. 4

recommended in [5]. Let us note that a constant value of Pr_T was used in the original method [6].

However, neither the application of (1) nor the increase, recommended in [4], in the number of points of the computational mesh in the viscous sublayer resulted in any substantial improvement in the results. Analysis of the nature of the discrepancy between the results of the computation and the experiment permitted the assumption that the reason is the formulation, proposed in [6], of the boundary condition for the heat-transfer equation at the wall. To simplify the computations near the wall, a one-dimensional Couette flow model was taken in which the convective heat transfer along x was neglected. The temperature gradient along the length should be taken into account for an abrupt change in the boundary conditions.

The dependences $C_f = C_f(Re_y)$ and $St = St(Re_y)$ were determined from the solution of the model problem [6] and were utilized in formulating the boundary conditions.

A formulation of the model problem with the component $\partial T/\partial x$ taken into account is proposed, wherein the heat transfer near the point of the abrupt change in the thermal condition can be described approximately by the equation

$$u\frac{\partial T}{\partial x} = \frac{\partial}{\partial y}\left(a_+\frac{\partial T}{\partial y}\right) \quad \left(a_+ = \frac{\nu}{\Pr} + \frac{\nu_T}{\Pr_T}\right)$$

with the initial condition $T(x_0, y) = T_e$ and the boundary conditions $T(x, \infty) = T_e$, $\partial T/\partial y|_{y=0} = -q_w/\lambda$ [or $T(x, 0) = T_w$] for $x > x_0$. The velocity profile u(y) was calculated from the analytic dependence [7]

$$y^+ = u^+ + \exp(-A)\left[\exp(Ku^+) - 1 - Ku^+ - \frac{(Ku^+)^2}{2!} - \frac{(Ku^+)^3}{3!} - \frac{(Ku^+)^4}{4!}\right],$$

where A = 2.05 and K = 0.41. Values of the turbulent viscosity ν_T found from the formula $\nu_T = (0.435y)^2\left|\frac{\partial u}{\partial y}\right|[1 - \exp(-y^+/A)]^2$ and Pr_T calculated from (1) were taken to calculate the effective thermal conductivity.

The dependence $St = St(Re_{\Delta x})$ [$Re_{\Delta x} = u_\tau(x - x_0)/\nu$] represented in Fig. 2 in the form of the function $\log St$ of $\log Re_{\Delta x}$ for two versions of the change in the thermal condition on the wall was determined from the solution of the modified model problem (the points are the numerical solution and the lines are the parabola approximation). It is seen from the graph that the curve for a given flux (line 1) passes above the curve for a given wall temperature (line 2). Moreover, for the first case the abscissa of the emergence of the curve "on the shelf" is greater than for the second. This latter means that the zone of action of the correction function for a given temperature is less than for the given flux.

The numerical results obtained were approximated by the analytical expression $St(Re_{\Delta x}) = 10^{f(Re_{\Delta x})}$. Here $f(Re_{\Delta x}) = p_0 + p_1(\log Re_{\Delta x}) + p_2(\log Re_{\Delta x})^2$; $p_0 = -1.3738$, $p_1 = -0.5609$, $p_2 = -0.06305$, $0 \le Re_{\Delta x} \le 30{,}000$ for the given flux, while $p_0 = -1.4044$, $p_1 = -0.535$, $p_2 = 0.0622$ for a given temperature.

The approximation in the Patankar–Spalding program was introduced in the form of the correction function $\varphi(Re_{\Delta x}) = St(Re_{\Delta x})/St_0$, where St_0 is the value of the Stanton number at the point $x = x_0$, evaluated according to the standard dependence $St_0 = 0.0252/Re_T^{**0.25}$.

The results of computations performed by using the correction introduced are represented in Figs. 3 and 4. Shown in Fig. 3 is the change in the Stanton number along the heat-transfer section for u_e = 51, 30.1, 84.35, and 84.35 m/sec; T_e = 21, 24.3, 22.1, and 22.1°C; and q_w = 6145, 9813, and 17,550 W/m²; T_w = 100°C (lines 1-4, dashed lines are the computation by the Patankar–Spalding method; the continuous lines are by the same method with the correction function $\varphi(Re_{\Delta x})$; 1 is experiment [3], and 2 and 3 are [2]. The notation in Fig. 4 is the same as in Fig. 3. As is seen, the agreement between computation and experiment has improved considerably.

Because of the inaccessibility of experimental data for a "jump" in the surface temperature to the authors, a test problem analogous to that represented in Fig. 1 was solved. The surface temperature measured in an experiment [2] (line 3 in Fig. 4) was used as thermal boundary condition. As is seen from Fig. 3, the Patankar–Spalding method describes the heat-transfer process near the section with the "jump" poorly even for this boundary condition. Introduction of the correction function analogously to the case of a given thermal flux results in improvement of the agreement between the computation and the experiment.

Therefore, taking account of the longitudinal convective transfer in the formulation of the boundary condition permitted computation of the heat transfer near the section of the discontinuity in the thermal state of the surface with good accuracy.

The Patankar–Spalding program with the correction function inserted can be used to solve analogous problems.

LITERATURE CITED

1. E. R. Eckert and R. M. Drake, Theory of Heat and Mass Transfer [Russian translation], Gosenergoizdat, Moscow–Leningrad (1961).
2. V. N. Vasechkin, B. P. Mironov, and N. I. Yarygina, "Influence of flux prehistory on the turbulent heat-transfer process," Turbulent Boundary Layer under Complex Boundary Conditions [in Russian], Inst. Thermal Physics, Sib. Branch, USSR Acad. Sci., Novosibirsk (1977).
3. Hartnett, Eckert, and Birkebach, "Analysis of fundamental characteristics of a turbulent boundary layer with air delivery through tangential slots," Heat Transfer [Russian translation], 83, No. 3 (1961).
4. Brown and Antonia, "Analysis of the turbulent boundary layer behind a section with a jumplike change in surface temperature," Heat Transfer [Russian translation], 101, No. 1 (1979).
5. P. N. Romanenko, Hydrodynamics and Heat and Mass Transfer in the Boundary Layer. Handbook [in Russian], Énergiya, Moscow (1974).
6. S. Patankar and D. Spalding, Heat and Mass Transfer in Boundary Layers [Russian translation], Énergiya, Moscow (1971).
7. Dean, "A single formula for the complete velocity profile in a turbulent boundary layer," TOIR [Russian translation], 98, No. 4 (1976).